Engineering Notes

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Flexible Spacecraft Control Design Using Pole Allocation Technique

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Introduction

CURRENT attitude control design procedure in dealing with multiloop coupled flexible spacecraft employs classical control design approaches to arrive iteratively at satisfactory controllers.[1,2] This process is indirect and can become very tedious for spacecraft systems with a large number of control loops and/or having groups of structural frequencies clustered within some narrow regions in the frequency spectrum. Optimal control techniques,[3,4] on the other hand, can simultaneously provide control laws for all loops upon solving the matrix Riccati equation for a controllable system. One problem associated with this approach is that a designer has no direct control over the closed-loop system eigenvalues. This Note proposes a design approach through preassigning poles, allowing an analyst to have direct control over the closed-loop system eigenvalues.

The algorithm presented in Refs. 5-7 admits incomplete state feedback and was adopted in the present study for the following system:

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

$$u = Ky \tag{3}$$

where x, u, and y are the system state, input, and output vector, of dimensions n, r, and m, respectively. A, B, C, and K are constant matrices. The pole allocation process is accomplished by initially assigning max. (m, r)[6] poles, after which a coordinate transformation takes place and the remaining poles can be placed.[6,7] More details on this subject can be found in Ref. 8. A computer program incorporating this procedure was developed and was applied to the spacecraft shown in Fig. 1.

The idealized system consists of a rigid central body B^* with three reaction wheels (not shown in the figure) aligned with the spacecraft yaw, roll, and pitch axes for its attitude control. On the far end of a flexible boom, a flexible solar array A^* is driven by a motor under closed-loop control to track the Sun. $\boldsymbol{b}_i$, $\boldsymbol{n}_i$, and $\boldsymbol{a}_i$ $(i = 1,2,3)$ are mutually perpendicular, right-handed unit vectors attached to B^*, N, and the elastically undeformed A^*, respectively. It is not difficult to appreciate the complexity in designing a control subsystem for this spacecraft. The solar array assembly is very flexible and has low structural frequencies, which can interact with the control system. The asymmetric configuration results in large products of inertia which, in turn, will introduce cross coupling among the controllers. Compounding these problems, the system flexibility and inertia characteristics are modulated as the array rotates around the shaft. The control design effort for such a system can be greatly simplified using the pole allocation approach.

System Equations

The nonlinear system dynamical equations were derived using the "hybrid-coordinate" formulation.[9] Linearization in the variational coordinates of these equations with respect to an inertially stationary nominal state‡ leads to the following set of matrix differential equations:§

$$a^{\omega}\dot{\omega} + a^{g}\omega + a^{\varphi}\ddot{\varphi} + a^{\epsilon}\ddot{\epsilon} + a^{\eta}\ddot{\eta} = T^{b} \tag{4}$$

$$b^{\omega}\dot{\omega} + b^{\varphi}\ddot{\varphi} + b^{\eta}\ddot{\eta} = T_{\varphi} \tag{5}$$

$$c^{\omega}\dot{\omega} + c^{\varphi}\ddot{\varphi} + c^{\eta}\ddot{\eta} + c^{D}\dot{\eta} + c^{K}\eta = F^{\eta} \tag{6}$$

$$a^{\epsilon}(\dot{\omega} + \ddot{\epsilon}) = T^{\epsilon} \tag{7}$$

The 3×1 vector ω contains three $\boldsymbol{b}_i$ components representing the deviations of the inertial angular rates of B^* with respect to the nominal state, the scalar φ and 3×1 vector ϵ denote the deviations of the array and the wheel motion, respectively; and η of dimension J is the appendage modal vector for the flexible solar array. More background discussion of the spacecraft can be found in Ref. 2.

In the subsequent analysis, external torques are ignored and the wheel spin inertias a^{ϵ} are assumed small. Let θ_1, θ_2, and θ_3 be a 1-2-3 set of the central body attitude angles. Equations (4-7) can be converted into a system of first-order equations exactly in the form defined by Eq. (1). For convenience of discussion and presenting results, x is partitioned into two parts, i.e.,

$$x = \begin{Bmatrix} x^1 \\ x^2 \end{Bmatrix} \tag{8}$$

where

$$x^{1^T} = (\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3, \dot{\varphi}, \theta_1, \theta_2, \theta_3, \varphi) \tag{9}$$

$$x^{2^T} = (\dot{\eta}_1, \dot{\eta}_2, \cdots, \dot{\eta}_J, \eta_1, \eta_2, \cdots, \eta_J) \tag{10}$$

The state vector x has dimension $(8 + 2J)$. The first eight states (x^1) are normally measurable quantities while the remaining states (x^2) are not traditionally sensed parameters. The system equations in parallel to Eqs. (1-3) can now be ex-

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‡The reaction wheels can have constant spin rates relative to the central body.

§Superscript will be used throughout this Note as part of the notation. Powers will be identified by numbers outside parentheses, i.e., $(\ )^2$, $(\ )^4$, etc.

pressed as,

$$\begin{Bmatrix} \dot{x}^1 \\ \dot{x}^2 \end{Bmatrix} = \begin{bmatrix} A^{11} & A^{12} \\ A^{21} & A^{22} \end{bmatrix} \begin{Bmatrix} x^1 \\ x^2 \end{Bmatrix} + \begin{bmatrix} B^1 \\ B^2 \end{bmatrix} u \tag{11}$$

$$\begin{Bmatrix} y^1 \\ y^2 \end{Bmatrix} = \begin{bmatrix} C^{11} & C^{12} \\ C^{21} & C^{22} \end{bmatrix} \begin{Bmatrix} x^1 \\ x^2 \end{Bmatrix} \tag{12}$$

$$u = \begin{bmatrix} k^{11} & k^{12} \\ k^{21} & k^{22} \end{bmatrix} \begin{Bmatrix} x^1 \\ x^2 \end{Bmatrix} = kx \tag{13}$$

where

$$k = KC \tag{14}$$

and u is a 4×1 vector containing the four control torques, i.e., T^ϵ and $T\varphi$. Equation (13) defines control laws for the system. The feedback gain matrix k is generated from placing desired poles.

Results and Discussions

Assuming that the three wheels nominally have zero spin rates so that $a^g = 0$, the open-loop system has eight zero eigenvalues which create numerical difficulties in the pole allocation process. Furthermore, the system distribution matrix A, in general, is not well conditioned because a^ω is considerably larger in magnitude than a^η or c^η. The first problem arising from multiple zero eigenvalues can be overcome by arbitrarily selecting an initial feedback gain matrix that makes all closed-loop system eigenvalues nonzero. Pole allocation can then proceed from there. The condition of A can be improved through scaling the appendage mode shapes ϕ^i where superscript i indicates mode number. To simplify manipulations, a constant scale factor f is assumed for all modes. For the spacecraft under consideration, f was chosen to be 30, resulting in modal masses comparable to the array inertia about its boom axis.

Case I

In this example, the array flexibility is represented by its fundamental appendage mode at a frequency of 4.18 rad/s. The damping ratio was chosen at 0.002. This mode involves the bending of the boom and the array mast with a deformed array oscillating about the boom axis as well as in the $\pm a_2$ direction. The nonzero structural system frequency is 5.65 rad/s. The state vector x has a dimension of 10 ($n = 10$). The dimension of u (i.e., r) is 4. Choose output vector y to be y^1 in Eq. (12) so that

$$C^{11} = E$$

$$C^{12} = 0$$

where E is the 8×8 identity matrix. The dimension of y (i.e., m) is therefore 8.

Theorem 1 in Ref. 7 states that a number of min. (n, $m+r-1$) closed-loop system poles can be arbitrarily assigned. This suggests that 10 poles can be assigned in the present example. They are listed as follows: -0.1, -0.125, $-0.1 \pm 0.03i$, $-0.032 \pm 5.65i$, $-0.09 \pm 0.015\,i$, $-0.001 \pm 0.05\,i$. Note that the passive structural system frequency of 5.65 rad/s is preserved. The feedback gain matrix k resulting from placing these poles is independent of η_l and $\dot{\eta}_l$.

A time simulation run with $\theta_l(0) = 0.1$ deg over 650 s was performed. The last pair of the poles, i.e., $-0.001 \pm 0.05\,i$, would dominate the long-term behavior because it has the longest time constant as verified by Fig. 2. Transient effects are apparent during the first 100 s. For the remaining 550 s, the simulation exhibits an oscillatory motion with a period of 125 s and a time constant approximately 1000 s. Both the period and time constant check extremely well with the last pair of poles.

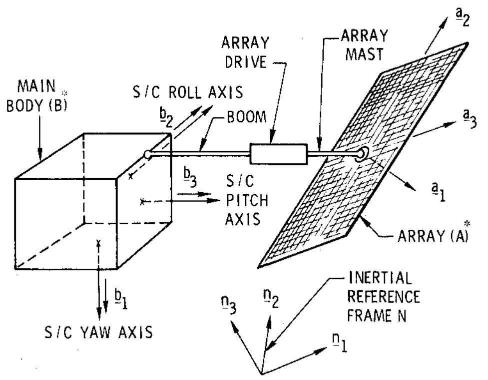

Fig. 1 Idealized spacecraft.

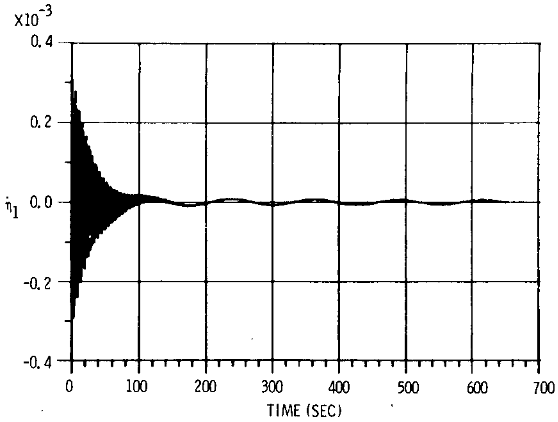

Fig. 2 η_l vs time for 650 s.

Case II

The second appendage mode is now added to the analysis. The mode shape involves bending of the boom and the mast together with a symmetrically bent array about the a_3 axis. The state vector dimension is increased to 12, while the dimension of u still remains at 4. Since min. (n, $m+r-1$) poles can be arbitrarily placed, the dimension of the output vector y is selected to be 10 which includes η_l and $\dot{\eta}_l$. The dimensions of the system are, therefore, $n = 12$, $r = 4$, and $m = 10$.

The desired closed-loop poles are -1.0, -1.25, $-1.0 \pm 0.3i$, $-0.5 \pm 0.2i$, $-0.9 \pm 0.15i$, $-0.8 \pm 0.5i$, $-0.32 \pm 0.9i$. Note that the longest time constant among the closed-loop poles is $1/0.32 = 3.125$ s, indicating a fairly rapid attenuation of perturbations. Furthermore, the highest frequency among the desired poles is 0.9 rad/s which is considerably lower than the two structural system frequencies of 5.6 and 6.1 rad/s. This indicates that the control system has significantly altered the structural characteristics. These two factors are responsible for the high gains obtained for this case.

A serious difficulty in addition to the high gains in implementing this controller lies in its dependence on the appendage modal information, i.e., η_l and $\dot{\eta}_l$. To directly sense the modal coordinates would require a large number of sensing devices (e.g., accelerometers, strain gages, etc.) for complex, flexible appendages. An alternative to this is to develop a Luenberger observer[10] to estimate the required modal information. An important step in designing a Luenberger observer is to assign poles to meet a convergence criterion. The pole allocation process proposed here can also be used to design the observer which is beyond the scope of this Note.

The design approach proposed here allows an analyst to have direct control over the closed-loop system poles. Although it still requires extensive further investigation to solve many practical problems such as parameter sensitivity, modal truncation error, etc., the pole allocation process can be a viable preliminary control design tool. In view of the trend of modern spacecraft to carry onboard computers, the implementation of a multivariable controller/observer control subsystem to a complex flexible spacecraft becomes more physically realizable than ever before. The ease with which a set of linear controllers satisfying key performance specifications can be directly obtained will considerably simplify the preliminary control design effort for future spacecraft.

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Flat Spin Recovery of a Rigid Asymmetric Spacecraft

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Introduction

THE motion of any spacecraft having nonzero angular momentum will decay to flat spin – spin about the axis of maximum moment of inertia – in the absence of active attitude control. Techniques for flat-spin recovery, that is, for re-establishing desired spacecraft attitude, have been discussed for the spinner intended to spin about its axis of minimum moment of inertia,[1] for momentum bias spacecraft,[2,3] and for dual-spin spacecraft.[4]

The treatment in Ref. 1 of the recovery strategy employing spin-up thrusters for spinners, is, however, approximate and leads to an expression for the requisite recovery torque that, while suitable for the configuration then of interest, is inappropriate for the general asymmetric spacecraft. In this Note, flat-spin recovery for the spinner is addressed exactly, and a generally valid expression for the requisite spin-up thruster recovery torque is derived.

Analysis

The motion of a rigid asymmetric spacecraft initially executing pure spin about its axis of maximum moment of inertia and influenced by a torque $T(t)$ applied about its axis of minimum moment of inertia is described by

$$A\dot{\omega}_1 + (C-B)\omega_2\omega_3 = T(t) \tag{1a}$$

$$B\dot{\omega}_2 + (A-C)\omega_1\omega_3 = 0 \tag{1b}$$

$$C\dot{\omega}_3 + (B-A)\omega_1\omega_2 = 0 \tag{1c}$$

with initial angular rates

$$\omega_1(0) = \omega_2(0) = 0, \qquad \omega_3(0) = \Omega \tag{2}$$

and with the inequality on A, B, and C, spacecraft moments of inertia about body-fixed axis 1, 2, and 3, respectively

$$C > B > A$$

Manipulation of Eqs. (1b) and (1c) leads to an integral of motion

$$s_2\omega_3^2 + s_3\omega_2^2 = s_2\Omega^2 \tag{3}$$

where

$$s_2 = (C-A)/B, \qquad s_3 = (B-A)/C$$

The form of Eq. (3) suggests natural substitutions

$$\omega_2 = \Omega\sqrt{s_2/s_3}\,\sin\eta, \qquad \omega_3 = \Omega\cos\eta \tag{4}$$

In either Eq. (1b) or (1c), substitutions (4) yield

$$\omega_1 = \dot{\eta}/\sqrt{s_2 s_3} \tag{5}$$

Substitutions (4) and (5) together with

$$\phi = 2\eta \qquad \tau = \sqrt{s_1 s_2}\,\Omega t \tag{6}$$

where

$$s_1 = (C-B)/A$$

transform Eq. (1a) into

$$\phi'' + \sin\phi = \kappa(\tau) \tag{7}$$

where the prime denotes a differentiation with respect to τ and where

$$\kappa = 2T/(s_1 A\Omega^2)\cdot\sqrt{s_3/s_2} \tag{8}$$

Initial conditions associated with Eq. (7) may be developed readily from the initial angular rates (2)

$$\phi'(0) = \phi(0) = 0 \tag{9}$$

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